CRISS IMPROVES MATH COMPREHENSION

Create and Evaluate Problems

Sue Harding, in Kalispell, Montana, knows that her fourth graders become more proficient with math problem solving, if they have many opportunities to write and then evaluate their own problems. Each morning she greets her students with a number on the board. Today they see the number 9. While Sue takes role and counts lunch money, her students take out their math journals and begin to derive as many problems as they can that have 9 as the answer.

Matthew writes:

5 + 4 = 9
(10 - 2) + 1 = 9
20 - 11 = 9
18 g 2 = 9
3 x 3 = 9

Matthew then chooses one of his problems and writes a story problem:

Story problem for 9. I had 20 pumpkins all cut out with scary faces. On Halloween, before I went trick or treating, I set out my 20 pumpkins on the porch. I went trick or treating for 2 hours. Then I went to a Halloween party for 3 hours. I was out for five hours total. When I went home, 11 of my pumpkins with scary faces were gone. How many pumpkins do I have left? 20 - 11 = 9.

Next, Matthew meets with Vanessa to read and to talk about their story problems. Sue gives each pair of students a Problem-Solving Checklist to guide their evaluation. Sue asks, "Can you solve one another's problems? How might you make your problems clearer? Do your problems contain the key parts of a story problem?" Matthew and Vanessa read through their problems and evaluate their work using the checklist as a guide.

Problem-Solving Checklist

1. Does the problem contain a clear question? Will the reader know what to do?
2. Does the author present facts clearly?
3. Does the author give you clues as to what operation you are supposed to use?

After students complete any revisions, Sue asks for two volunteers to make a transparency of their problems for the whole class to solve.

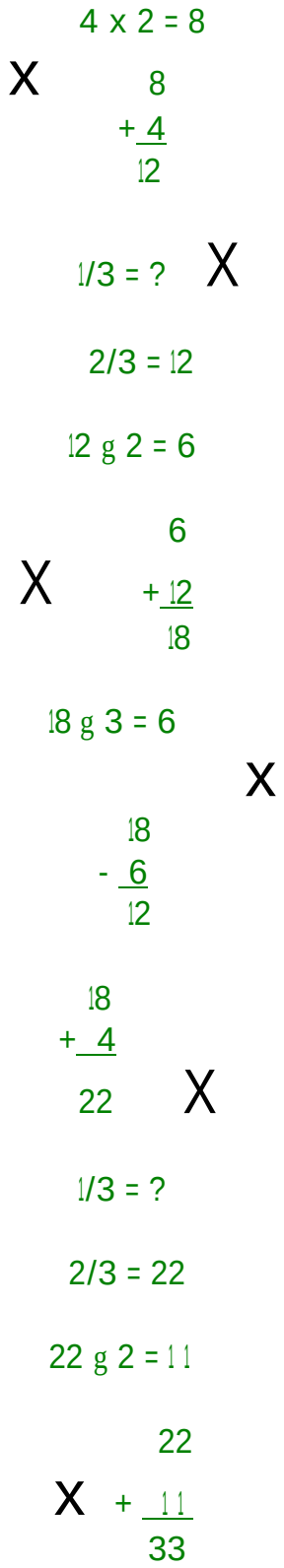
Writing Aids Problem Solving

Staci Auck, a Kalispell, Montana, seventh grade math teacher, asks students to write frequently in her classroom. She believes that writing helps students think through difficult steps in problem solving. Although she wants her students to primarily write for themselves, she also finds that their writing provides her with valuable insights about the students' thinking. Writing also encourages students to come up with a variety of creative ways to solve the same problem. Having students share their writing and thinking with each other assists them in discovering several options for solving a particular problem. In one lesson, she presented students with the following problem:

Last summer we went camping in Yosemite, and the first night we did a dumb thing: we left our food on the ground. A bear came along and ripped up one third of our total number of dried meals. The next day we ate four of the meals and tied the rest of the food up in a tree. It didn't seem to help, because one third of the meals we had left was ripped open by another bear. During our third day, we ate four more meals and that night, despite everything we did, one half of the remaining dried meals was ripped apart. We gave up, ate the four remaining dried meals, and headed home.

Can you tell how many dried meals we started with?

Brad reads through the problem several times. Then, he begins to write. As he writes, he begins solving the problem numerically. He goes back and forth from writing down numbers to writing down words.



To get your answer, you have to start at the bottom. It says that half of the remaining meals was eaten to give the 4. So you have to multiply by 2 because a half is equal to 2. This gives you 8. Plus the 4 more meals they ate gives you 12. Now if 1/3 of the number before this was ripped apart, that means that they have 2/3 of that number left. So, 6 is half of the 2/3 left which is 12. So you have to add 6 to 12 to get 18. Let's double check. If you minus 1/3 from 18, it gives you 12. So, that's right. Then they ate 4 more meals to give them 22 meals. The next step you do is the same as the last. 1/3 was ripped up of the number before this. You have to find out what half of 22 is, because 22 is equal to 2/3. And that's how much is left. The number is 11. So add 11 to 22 and it gives you 33. That's your answer.

As Brad completes the problem, he turns to Mark, his math partner. They talk through their problem strategies together and share their work. Brad's writing makes visible problem-solving strategies that would otherwise remain hidden. It also allows him to represent his own thinking so that he can monitor his progress.

Analyzing Math Errors Through Writing

Beverly Krusz, an eighth grade algebra teacher in Juneau, Alaska, sent us a story from her classroom.

> *In algebra and pre-algebra, I have always expected my students to check and correct their work by redoing problems they missed. For some, this involved doing an analysis of their mistakes; but, for most, it meant a shrug of, "Oh, well, I missed five problems. Guess I'll do them over and hope they're right." They were reluctant to change this habit because of the time involved in analyzing their errors. Now I have tried something different. I quickly check their tests and quizzes and then give students an opportunity to raise their grades. They have to circle their specific errors on the test in red. Then, on a separate piece of notebook paper, they write in words what they did wrong and explain how to correct it. They also have to redo the problems correctly. Then, the students staple the explanation paper to the test and submit the work to me again for an adjustment on their grades.*

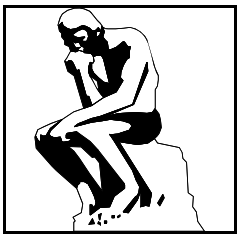
Example: Test problem:

Solve for x and y: 3x - y = 8; y = x - 2

Student's original incorrect answer on test:

3x - y = 8	- -3 + y = -2
-x + y = -2	-3 -3
2x = -6	y = -5
x = -3	(-3, -5)

Student's correction:



3x - y = 8 -3 + y = -2
-x + y = -2 +3 +3
2x = 6 y = 1
x = 3 (3, 1)

When I added -2 to 8, I got -6 instead of 6 so it made both parts of the answer wrong.

Bev asked her students to react to this process of identifying and correcting their own errors, using a "fast write."

> I like the new way of doing the tests; even though it takes a little longer. I like that we can redo the problem and learn how to do it and what I did wrong. The old way I really didn't know how to do it any better for the next time, nor did I get any better grade.

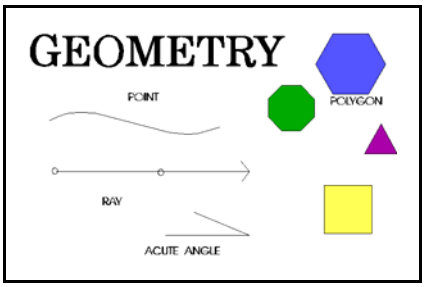
> I like the way we do it now. You have to look back into the problem and see what you did wrong. Many times I see my mistake (many times dumb ones) and I try to be more aware of those kinds of problems or things in upcoming problems. I think people will learn more with the way we do it now.

Bev adds:

Having students write explanations does take time for me to read, but I feel it is worth it. A side benefit is the math discussions that occur among the students. I organize my students into groups of four. When returning their papers, I don't provide them with a list of answers so there is a lot of comparing of answers and work among members of the group. This is especially helpful to the student who skipped a problem originally because he didn't understand it. Now this student has to learn how to do it in order to write the explanation. Not only does the learner benefit, but so does the student who explains it.

Creative Writing and Student Portfolios

Marilyn Cavanna's College Prep geometry class (Manchester, Connecticut) sent us some delightful writing samples. Marilyn writes that most of her student work is in the form of writing samples. Her students keep math portfolios. In their portfolios, students explain concepts from their math text. Marilyn requires that the students write why they picked certain problems or proofs to include. As part of the portfolio, she encourages them to include RAFT papers and poetry. For the RAFT papers, she had her students take something they had learned in geometry and write about it to an audience of elementary students.



> Once upon a time, there was a curious and perfectly shaped equiangular triangle. I call him "Bob". Bob was wandering around an abstract math wonderland called Cavannaland, a land perfect for such a sojourn. Bob came across a small cottage with smoke rolling out of a brick chimney and decided it would make a great place to spend the evening. Bob had always wondered what type of triangle he was, but because of the scarcity of such shapes in Cavannaland, he was never told. Bob let himself in and looked around, but there was no one around. He decided to examine the bedroom for a place to sleep. In it he saw six beds set in a circle. Bob tried to sleep in one of the beds and it was quite uncomfortable. A narrow angle at one end hurt his

sensitive 60 degree angle. "This must be an acute triangle bed because all of its angles are acute and one is less than my angles." The next bed looked like half of a square because it had a right, 90 degree angle on one side. This wouldn't do either. The bed marked obtuse had one uncomfortable angle which measured 160 degrees. Maybe the other beds might be suitable. Bob knew, because of a theorem he once knew, named Theorem 20, that because his angles were all the same size, that all of his sides must be equal in length. (Bob always wondered why his body was always marked with arcs and tick marks.) The fourth had no sides congruent and was meant for a scalene triangle, not him. The fifth bed had a familiar name to it, "Isosceles". Bob had been called this before by a fat, wart-cornered Ogre who was a 12,762-gon! He tried the bed, but the vertex angle was too narrow and the base was too short. He had almost given up hope, when he finally spotted a perfect-looking bed marked "For Equis Only". "That's what I am! I'm an equiangular, equilateral triangle!" Bob was glad to discover the truth on his own.

Marilyn's students also wrote poems about their favorite geometry theorem. An example of the poems appears here.

MY FAVORITE THEOREM
by Tom Breen

'Twas a thick and slothful early morn,
when Maude and I set out to Innisfree,
past the darkened Georgian homes,
with their crumbling outer walls.

To Tara;--High Hill of Tara! seat of high-born kings
ancient place of knowledge and divining arcane things--
was where our carriage sped this day,
for knowledge it was that which we sought,
to be attained without delay.

For a vile and grievous dilemma,
had come to hang above us,
a preponderance to which our well-bred minds,
could achieve little but naught.

Maude and I were children of letters,
the written word and not the number.
And the problem which had beset us this day
was math, and thus to us foreign.

We had been charged to discover,
the perpendicular bisector
of a given triangle,
which had us so confused.

So we climbed the Hill of Tara,
desolate these thousand years,
and pitched five bright and golden coins,
into the ruined Well of Knowledge there.

We called out into the inky well,
in the manner of Fionn and Boru.
And asked our burning query,
then waited, quiet and still.

Soon a voice primordial,
and speaking the tongue of the Gaels,
answered us with a riddle;
"Two points equi imply a perpendicular bisector."

At first, we were surprised,
and quiet with reverent awe.
But then, sweet joy set in:
our math problem was solved!

We drove back to Dublin,
with a haste born of glee.
Bringing a new theorem,
a gift from another time.

Inspired by William Butler Yeats